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The acceptability of papers offered must largely depend upon the judgments of our editorial referees. Final decisions will be made as promptly as possible and reported to those concerned.

This Journal is dedicated to the following aims:

1. THROUGH PUBLISHED STANDARD PAPERS ON THE CULTURE ASPECTS, HUMANISM AND HISTORY OF MATHEMATICS TO DEEPEN AND TO WIDEN PUBLIC INTEREST IN ITS VALUES.
2. TO SUPPLY AN ADDITIONAL MEDIUM FOR THE PUBLICATION OF EXPOSITORY MATHEMATICAL ARTICLES.
3. TO PROMOTE MORE SCIENTIFIC METHODS OF TEACHING MATHEMATICS.
4. TO PUBLISH AND TO DISTRIBUTE TO THE GROUPS MOST INTERESTED HIGH-CLASS PAPERS OF RESEARCH QUALITY REPRESENTING ALL MATHEMATICAL FIELDS.

Forthcoming Analysis of Survey Returns

One can hardly question that the secondary mathematics survey returns gathered from the offices of forty-eight state superintendents of education and published in two installments in this Magazine will have great interest and value for every serious-minded teacher of mathematics. The reactions to our questionnaire were straightforward and without bias. Much of their significance for the teacher will lie in the fact that they are statements of practical educational administrators.

The indications which are furnished by the returns concerning future trends of secondary mathematics should be discerned by any one who cares to study thoughtfully the data here presented. Nevertheless propriety seems to demand that our own formulation of such analysis be presented for the accommodation of the readers of this journal. It is hoped that we shall be able to publish this analysis in the December issue.

*Survey of Present Status of Secondary Mathematics in the United States

By S. T. Sanders

Last spring the following three questions were sent out to forty-nine State Superintendents of Education including the Superintendent of the District of Columbia.

1. Has Mathematics as a required subject for graduation from high schools been eliminated in your state?
2. If it has not been eliminated as a requirement, is there a definite prospect that it will be eliminated at an early date?
3. In your judgment, are there good reasons for the hope that mathematics will have increased use in the secondary schools of your state?

In our October issue were published 47 replies to question 1. Forty-eight had actually been returned but, due to a very much-regretted unexplainable oversight, the 48th was not included in the published list.

To the question,

> "Has mathematics as a required subject for graduation from the high schools been eliminated in your state",

Mr. Webster H. Pearce, Superintendent of Public Instruction had sent in the answer from

MICHIGAN

"No. A pupil may graduate from high school without the mathematics requirement. He then would not be permitted to enter university or college."

Because of the close interrelations of question 2 and question 3, and the further fact that a "no" answer to question 1 would make answer to question 2 unnecessary, we are publishing under each named state both answers to these questions, namely:

2. "If mathematics has not been eliminated as a high school requirement in your state is there a definite prospect that it will be eliminated at an early date?"
3. "In your judgment, are there good reasons for the hope that mathematics will have increased use in the secondary schools of your state?"

* Editorial interpretation of the returns from this survey will be made in a later issue.

ALABAMA

"I see no indication that there will be any change in the position now held by mathematics in the Alabama plan. We have elementary algebra offered as an elective in the ninth grade, or third year of the junior high school. In the first year of senior high school, or the tenth grade, plane geometry is offered. A semester in solid geometry and a semester in intermediate algebra may be taken either in the eleventh or twelfth grade. In the smaller schools this may be alternated. In some of the larger schools trigonometry is offered in the last or twelfth grade.

"In some of the city schools a unit and a half of elementary algebra is required instead of the unit which our state plan calls for. In these cases they probably include in this unit and a half of elementary algebra some of the work in intermediate algebra covered in our advanced elective in that subject."—W. L. Spencer, Director of Secondary Education.

ARKANSAS

"Opinion differs among competent authorities as to the importance of mathematics in secondary school curricula. From my point of view, it is inconceivable that mathematics should be eliminated from the secondary school curriculum. This is not to say, however, that the content of the courses in mathematics should remain as it is, or that the organization of the courses should not be radically changed. The progress of civilization has been made pretty largely as a result of quantitative thinking, and mathematics is the principal subject contributing to quantitative thinking."—M. R. Owens, State High School Supervisor.

ARIZONA

"There is no prospect that mathematics will be eliminated, nor are there reasons for hoping that it will have increased use in the secondary schools."—H. E. Hendrix, State Superintendent of Public Instruction.

CALIFORNIA

"We are in the midst of a program of reorganization and the general directions of reorganization in individual subject fields have not been fully determined. It is probable that there will be an increased emphasis upon the more practical type of mathematics instead of the ordinary traditional type, except for certain students who may need the traditional type of mathematics to meet technical requirements in professional courses."—Vierling Kersey, Superintendent of Public Instruction.

CONNECTICUT

"Our high schools have now a very large number of pupils who are not making preparation for college or employment in lines where formal mathematics is a requirement. I anticipate, therefore, that the importance of mathematics of the academic type will decrease in our secondary schools."—E. W. Butterfield, Commissioner of Education.

DELAWARE

"There is no prospect that mathematics will be eliminated, nor are there reasons for hoping that it will have increased use in the secondary schools."—H. V. Holloway, State Superintendent of Public Instruction.

FLORIDA

"There is no prospect that mathematics will be eliminated. There are grounds for the hope that it will have increased use in our secondary schools."—M. R. Hinson, State Director of Instruction.

GEORGIA

"I am of the opinion that mathematics will have increased rather than diminished use in the schools of this State."—M. D. Collins, State Superintendent of Schools.

ILLINOIS

"There is a tendency right now, in some of the smaller schools particularly, to limit the amount of mathematics required for graduation. I have not seen any tendency to increase mathematics requirements in the high schools of Illinois."—Harry M. Thrasher, State High School Supervisor.

INDIANA

"So far as I can see, there is no tendency in our state to eliminate the work in mathematics. We are trying to get up a course of study which will meet the needs of all types of students and provide for those who cannot do all in mathematics courses.

"All high school courses conform pretty closely to college entrance requirements and we feel that our students should have a fair training in mathematics."—V. R. Mullins, Director, School Inspection Division.

IOWA

"While two years of mathematics are at present required for college entrance in most of the Iowa colleges, there is a possibility that this requirement will be dropped in the near future."—R. C. Williams, Director of Research.

KENTUCKY

"It is not proposed to eliminate the requirement of one unit. Personally, I think as a result of better teaching and better textbooks that mathematics will have increased use in secondary schools of this state."—Mark Goodman, Public School Supervisor.

LOUISIANA

"There is no reason to believe at present that algebra as a required subject for graduation will be eliminated from the course of study either in the early or remote future.

"There are no reasons before us at this time to justify the belief that mathematics will have either an increased or a diminished use in the secondary schools of our State."—Chas. F. Trudeau, State High School Supervisor.

MAINE

"In the general or civic curriculum there is a definite trend toward a general mathematics of the sort typified by the Prentiss-Hall book "New Applied Mathematics."

"Personally I believe that this trend is in the right direction. Far too much time has been wasted in hopelessly trying to teach half of our students two years of algebra, geometry, and similar subjects when that time could much better be utilized

in giving citizenship subjects such as civics, economics, sociology and the histories, together with environmental studies such as general science and biology."—Harrison C. Lyseth, Agent for Secondary Education.

MASSACHUSETTS

"I think many of our high schools no longer require mathematics for graduation and there seems to be an increasing trend in this direction. Many schools which still do require mathematics, give an option between algebra and what is known as general mathematics, thus taking care of many pupils who cannot do the traditional type of algebra successfully."—Jerome Burtt, Supervisor of Secondary Education.

MICHIGAN

"The course of study in secondary mathematics needs revamping rather than eliminating."—Webster H. Pearce, State Supt. of Public Instruction.

MINNESOTA

"There are good reasons for the hope that mathematics will have increased rather than diminished use in the secondary schools of Minnesota."—E. M. Phillips, Commissioner of Education.

MISSISSIPPI

"It is my judgment that schools that do not require mathematics will increase in number rather rapidly."—S. B. Hathorn, State High School Supervisor.

MISSOURI

"There is no indication that the requirement of one unit of mathematics will be changed."—J. R. Scarborough, Director of High School Supervision.

MONTANA

"There is no prospect that mathematics will be eliminated at any early future time. Continued use of it is urged."—Miss Elizabeth Ireland, State Superintendent.

NEBRASKA

"I should not be surprised but that mathematics will be dropped as a specific requirement for admission, and that very shortly. This coming year, the University is carrying on an experiment and is intending to offer a course in mathematics to be taken by any student whether he has completed a mathematics requirement in high school or not.

"I think you will be very much interested in the writer's report on university and high school relations which appeared in the January, 1934, issue of North Central Association Quarterly. This study was made a year ago and reported at the meeting of the Association. You will find considerable discussion in this of the matter of subject matter requirements and various other problems relating thereto. I trust that you will feel perfectly free to write us further concerning this matter should you have any further question.

"In regard to your third question, I have every reason to believe that mathematics has a place in the secondary school curriculum and I would hope an increased rather than a diminished use. This, however, will depend upon the extent to which

mathematics instructors are able to relate the subject matter of mathematics definitely to the needs of those being trained to the end that they may more properly and effectively utilize the subject matter in their every-day experiences."—G. W. Rosenlof, Director, Secondary Education and Teacher Training.

NEVADA

"There is no prospect that mathematics will be eliminated from Nevada high schools. There will probably not be very much change in its present status."—Amy Hanson, Office Deputy.

NEW HAMPSHIRE

"It seems to me that the trend is to think of the needs of the individual pupil when determining whether or not he shall take mathematics. The adviser would thus consider the requirements of the particular field of work, in or out of school, which the pupil is to pursue. For this reason it is felt that mathematics beyond arithmetic is not necessary for certain fields, and the returns to certain individuals would be less than in some courses. Here in New Hampshire we try to emphasize the command of the fundamental principles of arithmetic, and give special drill to all pupils during their four years in high school when they do not meet the standard score in a standardized test on the fundamentals in arithmetic."—Russell H. Leavitt, High School Agent.

NEW JERSEY

"There is no prospect that we can see for such elimination in the near future.

"The answer to your third question is rather difficult to give in a brief letter. I think that our teachers of mathematics hope to develop more useful syllabus material in the near future and to make their important subject a more significant part of the high school course than it has ever been."—Howard Dare White, Ass't Commissioner of Education.

NEW MEXICO

"In my opinion mathematics will not be eliminated as a required subject for some time although the variety of mathematics offered in the high school has been cut down. Therefore, I do not believe you could say that mathematics has increased in the secondary schools."—Marianne Geyer, High School Supervisor.

NORTH CAROLINA

"There is no prospect of the elimination of mathematics from North Carolina high schools. The curriculum will be revised."—J. H. Highsmith, Director of Instructional Service.

NORTH DAKOTA

"From the statistics which we have compiled in this office it appears that the status of mathematics has remained approximately the same during the past ten years."—John A. Page, Director of Secondary Education.

OKLAHOMA

"There is some agitation for the elimination of mathematics as a required subject for a considerable number of the students in high school.

"I am of the opinion that there are good reasons for believing that geometry will be studied by fewer and fewer students in the high schools of this state. I expect to see a general course in mathematics introduced rather universally for the ninth grade for a large proportion of the high school students in the high schools of our state.

"We should like to know the results of your survey when such results are available."—J. Andrew Holley, Chief High School Inspector.

OREGON

"There is no definite prospect of its elimination, nor of an increased use."—C. A. Howard, State Supt. of Public Instruction.

PENNSYLVANIA

"The minimum requirement as listed in our Standards for the Classification of Secondary Schools is one unit in mathematics.

"I do not believe that there is any possibility of removing the mathematics requirement in the very near future.

"Indications from our Statistics Bureau are that enrollments in mathematics for each of the grades from 7 to 12 have steadily increased each year, extending over a period since 1928 for which data is available. The increase, however, may be due more to an increase in secondary school enrollment rather than in interest."—Walter E. Hess, Advisor, Secondary Education.

RHODE ISLAND

"We note no change, and no diminished emphasis on mathematics.

"We think you have reason neither for hope nor for despair. We think that the present tendency to develop mathematics as a useful study is very important, and that it suggests new possibilities."—Walter E. Ranger, Commissioner of Education.

SOUTH CAROLINA

"There is a tendency, I believe, on the part of several high school units of the State to eliminate mathematics as a local requirement, as has already been done by the State for the State high school diploma.

"I see no good reason for the hope that mathematics will have increased use in the secondary schools of this State."—John G. Kelly, State High School Supervisor.

SOUTH DAKOTA

"I believe that South Dakota will always require one year in mathematics. I can also report that approximately 75 per cent of the students graduating from our high schools now, are taking plane geometry."—R. W. Kraushaar, High School Supervisor.

TENNESSEE

"There is a probability that mathematics will be entirely eliminated in Tennessee as a required subject.

"There are no good reasons for the hope that mathematics will have increased use in Tennessee secondary schools. As a matter of fact, mathematics is tending to decrease in value in Tennessee. The social sciences for the past two or three years are securing more attention. I hope that these answers are the ones you are looking for."—R. R. Vance, Supervisor.

TEXAS

"It is my belief that a return of normal conditions will bring such a demand for technically trained people that mathematics in the secondary schools will have an increasing popularity."—M. B. Brown, Deputy State Superintendent.

UTAH

"I have no reason to believe that mathematics will have an increase rather than a decrease in use in the secondary schools of our state."—Charles H. Skidmore, State Superintendent of Public Education.

VERMONT

"At present there seems no definite prospect that the requirement of mathematics will be eliminated."—Charlotte M. Lowe, Secretary to the Commissioner.

VIRGINIA

"I do not believe that there is any definite prospect that elimination will be made in the early future.

"I find it difficult to answer your question relative to the increased use of mathematics in our secondary schools. While many of our schools at the present time offer four years of mathematics, I do believe that there is a greater tendency on the part of many principals to offer only three years of mathematics."—C. J. Hyslup, Acting Supervisor of Secondary Education.

WASHINGTON

"There is no prospect of the elimination of mathematics. There will only be a less number taking algebra and geometry. There will be a re-allocation of types of mathematics."—N. D. Showalter, State Superintendent of Public Instruction.

WEST VIRGINIA

"At the present time a committee is studying the curricula in West Virginia with the idea in mind of making recommendation as to what should and what should not be required. They will not be ready to make a report for some time and until they do there will be no change.

"In my judgment there are no good reasons for the hope that mathematics will be increased rather than decreased in the high schools of West Virginia. I believe that if any changes at all are made, it will be in removing the mathematics requirement."—A. J. Gibson, State Supervisor of High Schools.

WISCONSIN

"There does not seem to me any good reason for hoping that mathematics will have increased rather than diminished use in the secondary schools of Wisconsin. The college and university presidents of Wisconsin have recently agreed to eliminate it as a requirement for college entrance in the State."—J. T. Giles, State High School Supervisor.

WYOMING

"Most of our schools are so small in elective offerings that rarely does a student complete a full course without some mathematics. With the elimination of the former wider range of electives in our schools, it necessarily follows that mathematics is being chosen now more than before."—B. H. McIntosh, Commissioner of Education.

DISTRICT OF COLUMBIA

"I do not believe that mathematics will be eliminated. I believe that there are good evidences which would lead one to think that the study of mathematics will increase rather than diminish.

"I am of the opinion also that if there should be made a common sense, non-technical, and not over-psychologized consideration of the subject of mathematics we might be able to find some reasons for the taboo that seems to exist in the minds of people concerning mathematics."—S. E. Kramer, First Assistant Superintendent.

Some General Remarks Concerning Scientific Exposition

By Rudolph E. Langer

(Continued from the preceding issue)

As I try to think back to the precepts which my instructor in freshman English in college long ago sought to inculcate in me, I seem to recall that much emphasis was placed upon the tenet that every composition, whatever its nature, should be adorned by an introduction. If I may judge from the reactions observable in many of my own students, no requirement of composition is more onerous or inconvenient. It has become my conviction that the difficulty is largely ascribable to the fact that no other element of a composition is more inexorable in its demand for a true knowledge of the subject. A suitable introduction requires of the expositor an understanding, not merely of the subject facts themselves, but also of their historical genesis, their interdependence, and their relations to the great body of classical and generally known theory. The reader has a very just claim to a presentation of this material. Short though it may of necessity be, the skilled expositor will make of his introduction an element worthy of the space which it requires. Through it he will enable his reader to appraise in advance the significance and bearing of the subject at issue, and will seek his return in the enhanced interest of his work.

The body of the exposition will naturally contain all the elements of the subject to be set forth. It will resemble any other composition of comparable length in the matter of subdivisions into chapters, sections,

paragraphs, etc. If the work is a lengthy one and the reasoning intricate, the writer should avail himself at strategic points of opportunities to summarize briefly the partial results already attained, and to reestablish the outlook of the reader by reiterating the purposes and objectives still in view. Obvious redundance in statements is, of course, wasteful and tiresome, and so is to be carefully avoided. On the other hand, as in all didactics, repetition for emphasis must and should be resorted to when an occasion calls for the elaboration of a matter not entirely simple. The review of a fact from different viewpoints and its restatement in different terms are often most effective means for enriching the thought and helping the understanding. They are at the same time adopted to refresh and stimulate the interest of the reader.

The mode in which a subject is to be developed is, of course, the concern of the individual expositor, and will vary much with the subject to be presented. I shall spend but a few words upon this point, though indeed it largely determines the character of the entire composition. As sharply contrasted for the presentation of mathematical material, one finds the deductive and the inductive methods. Of these the former resorts characteristically to more or less bald statements of fact, often in the form of theorems, and follows these by presentations of evidence or chains of reasoning to substantiate them. It is the method familiarly illustrated by the "Elements" of Euclid. Its greatest merits are to be found in its conciseness and finish, in the positiveness of the results, and in the relative ease with which the bearings of the various hypotheses may be determined. The inductive advance into a subject, on the other hand, is, broadly speaking, one which in a measure follows the path by which the facts were or might have been originally discovered. In it the expositor allows his reader in a sense to discover with him. Heuristic considerations in which rigor may be tentatively ignored are often made to show the motivation of the various steps and processes, and the unfolding of the ultimate goal comes as a climax of the work.

Pedagogically the advantages of the inductive approach are patent. The great mathematician Euler was an inductive writer of the first order, and as you know he became in his time through his expository writings the mathematics teacher of all the world. He was wont to characterize a writer in the deductive style as one who covers up his own inductive ascent in order thereafter to speak like a God from the clouds. Perhaps the most serious defect of the inductive method of presentation is that of its lack of conciseness and finish. If used exclusively, it is apt to lengthen the discussion unduly, and to dilute the content so far as to introduce an element of tedium for the reader.

The philosopher Schopenhauer was a bitter opponent of mathematics, and in particular of the deductive mode of presentation as applied to it. Let me quote to you briefly from his writings on this point, though, as you will see, the sentiments are clearly those of an extremist. He says:

> "We ask that every logical proof shall be traced back to an origin in perception: but mathematics, on the contrary, is at great pains deliberately to throw away the evidence of perception which is peculiar to it . . . that it may substitute therefor logical demonstration. This must seem to us like the action of a man who cuts off his legs in order to go on crutches. We are compelled by the principle of contradiction to admit that what Euclid demonstrates is true, but we do not comprehend *why* it is so. We have therefore almost the same uncomfortable feeling that we experience after a juggling trick, and indeed most of Euclid's demonstrations are remarkably like such feats. The truth almost always enters by the back door, for it manifests itself *par accidens* through some contingent circumstance. Often a *reductio ad absurdum* shuts all the doors one after another, until only one is left through which we are therefore compelled to enter. Often . . . lines are drawn, we don't know why, and it thereafter appears that they were traps which close unexpectedly and take prisoner the assent of the astonished learner."

It is fortunate, of course, that neither of the two extreme methods which I have mentioned is forced exclusively upon the choice of an expositor. If skillful he will avail himself of both, discriminating in their use in such a way as to gain the maximum of effectiveness for his completed work.

There remains but one major point to which I still wish to turn your attention,—that of the use of words. In all scientific writing the language should be appropriate to the subject, that is, of a suitable dignity, direct, and devoid of inconsequential embellishment. General grammatical requirements are, of course, insistent, as they are in any work of composition. Even after they have been met, however, there are still the requirements of precision and effectiveness which in scientific work are of prime importance. When his subject is itself replete with intrinsic difficulties the expositor must seek by his use of language to set forth his facts with the utmost possible precision. His words must fit the ideas with exactness, neither so loosely as to be vague, which endangers losing the thought among the words, nor so tightly as to cramp the understanding of the reader by introducing additional perplexities. The discrimination between undesirable extremes and a

desirable medium which is adapted to the audience in view requires constant vigilance and deliberate attention. Shunning prolixity one should equally avoid the erroneous notion that the telegram is a model of conciseness, and that the mere excision of dispensable words is invariably desirable.

Not quantity alone, however, but quality as well calls for consideration in the use of words. Such quality is to be attained by discrimination and the fine choice of words to convey the ideas. A tolerably extensive vocabulary is on this account indispensable in the equipment of an expositor. Without it (to quote again) we are "like a bad cook who seizes the frying pan whenever he needs to fry, broil, roast or stew, and then wonders why all his dishes taste alike while in the next house the food is appetizing." As scientists we should cultivate language, the tool, with scrupulous exactness, nor react to carelessness in this respect with any more tolerance than we should condone in the use of any other instrument of precision.

I might perhaps add a word concerning the use of technical terms. These terms, often unintelligible to all but the specialist, are on the whole indispensable in scientific exposition. At times, however, I have felt that some scientists now and again do not remain entirely innocent of offense in the use of them; that they perhaps occasionally permit themselves to glory somewhat in the jargonistic character of their language to the extent of displacing an intelligible and adequate term by one intrinsically less desirable but better suited to convey surreptitiously a suggestion of the expositor's erudition.

In conclusion I will remark that as teachers the most of us are expositors by profession. It is our business to explain and expound, and to do so with the greatest effectiveness of which we are capable. As with all matters of skill the cultivation of expository power is in large measure a matter of patience, of industry, and of persistence. To produce an acceptable result most of us must be prepared to write and rewrite and rewrite again. "We must write with pains, that one may read with ease." As an authority for the statement that with sufficient effort we may generally find craftsmanship in this respect within our reach, I am going to close by quoting to you from a short essay entitled "Some Self-Discovered Canons of Effective Writing" by Huxley, one of the most noted of scientific expositors. He says:

> "I never had the fortune, good or evil, to receive any guidance or instruction in the art of English composition. . . . The business of a young writer is not to ape Addison or Defoe, or Gibbon . . . They were great writers, in the first place, because by dint of learn-

ing and thinking, they had acquired clear and vivid conceptions about one or the other of the many aspects of men or things. In the second place, because they took infinite pains to embody these conceptions in language exactly adapted to convey them to other minds. In the third place, because they possessed that purely artistic sense of rhythm and proportion which enabled them to add grace to force, and, while loyal to truth, make exactness subservient to beauty.

"I cannot say that these principles have been my own guides; they are rather the result of a long experience. A considerable vein of indolence runs through my composition, and forty years ago there was nothing I disliked so much as the labor of writing. It was a task I desired to get over and done with as soon as possible. The result was such as might be expected.

"If there is any merit in my English now, it is due to the fact that I have by degrees become awake to the importance of the three conditions of good writing which I have mentioned. I have learned to spare no labor upon the process of acquiring clear ideas—to think nothing of writing a page four or five times over if nothing less will bring the words which express all I mean, and nothing more than I mean; and to regard rhetorical verbosity as the deadliest and most degrading of literary sins. Any one who possesses a tolerably clear head and a decent conscience should be able, if he will give himself the necessary trouble, thus to fulfill the first two conditions of a good style. The carrying out of the third depends neither on labor, nor on honesty, but on the sense which is inborn in the literary artist."

Resultants and Symmetric Functions

By C. H. Sisam
Colorado College

The following proof that a symmetric function, with positive integer exponents, of the form

$$x_1^{i_1} x_2^{i_2} \ldots x_r^{i_r} \qquad i_1 \not< i_2 \not< \ldots \not< i_r$$

in the quantities $x_1, x_2, \ldots x_n$, can be expressed as a polynomial in the elementary symmetric functions

$$\Sigma x_1, \Sigma x_1 x_2, \quad \Sigma x_1 x_2 x_3, \text{ etc.,}$$

seems to be somewhat more easily understood, by students to whom the elementary properties of resultants have first been presented, than

the proofs of this theorem often found in the textbooks. In addition, it not only offers a direct method for actually finding the required expression for a given function, which is frequently easier actually to carry through than the methods usually suggested, but it also affords the student valuable practice in the manipulation of determinants and helps to enforce the meaning and essential properties of resultants.

Let $$f_1(x) = x^n + a_1x^{n+1} + a_2x^{n-2} + \dots + a_n$$

and $$f_2(x) = b_mx^m + b_{m-1}x^{m-1} + \dots + b_0$$

be two rational integral functions of x. We suppose that the roots of $f_1(x) = 0$ are $x_1, x_2, \dots x_n$, and that $m \nless i_1$. It will be noted that, for convenience in the following discussion, we have taken $a_0 = 1$ and the subscripts of the b_j in the reverse of the customary order.

By the resultant R, of $f_1(x)$ and $f_2(x)$, we mean the product

$$R = f_2(x_1) \,.\, f_2(x_2) \,.\, f_2(x_3) \,.\, \dots \, f_2(x_n) \tag{1}$$

so that R has the interesting property that it vanishes if, and only if, $f_1(x) = 0$ and $f_2(x) = 0$ have a common root.

We shall assume that the relations

$$a_1 = -\Sigma x_1,\ a_2 = \Sigma x_1x_2,\ a_3 = -\Sigma x_1x_2x_3, \text{ etc.} \tag{2}$$

have been established, so that our problem has been reduced to expressing the given symmetric function as a polynomial in the coefficients a_1, a_2, a_3, etc., of $f_1(x)$. It is in this form that we shall write our solution.

We also suppose that it is known that R can be expressed (by Sylvester's method) as a determinant of order $m+n$ in the coefficients of $f_1(x)$ and $f_2(x)$, namely, in the form

$$R = K \begin{vmatrix} b_m & b_{m-1} & b_{m-2} & \dots & b_0 & 0 & \dots & 0 \\ 0 & b_m & b_{m-1} & \dots & \dots & b_0 & 0 \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 & b_m & b_{m-1} & \dots & b_0 \\ 1 & a_1 & a_2 & \dots & \dots & a_n & 0 \dots & 0 \\ 0 & 1 & a_1 & \dots & \dots & \dots & a_n \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 1 & a_1 & a_2 & \dots & a_n \end{vmatrix} \tag{3}$$

wherein K is a constant different from zero.

Let us consider the quantities $x_1, x_2, \ldots . x_n$, and hence $a_1, a_2, \ldots . a_n$, as constants. From (1) and (3), we have, identically in the quantities b_j,

$$f_2(x_1) \cdot f_2(x_2) \ldots . f_2(x_n) \equiv K \begin{vmatrix} b_m & b_{m-1} b_{m-2} \cdot b_0 & 0 & \ldots\ldots & 0 \\ 0 & b_m & b_{m-1} \ldots b_0 & 0 \ldots\ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots\ldots 0 & b_m & b_{m-1} \ldots\ldots & b_0 \\ 1 & a_1 & a_2 \ldots\ldots\ldots & a_n \, 0 \ldots & 0 \\ 0 & 1 & a_1 \ldots\ldots\ldots\ldots & a_n \, 0 \, . & 0 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & \ldots\ldots\ldots 0 & 1 \, a_1 \, a_2 & \ldots\ldots\ldots & a_n \end{vmatrix} \tag{4}$$

Since this is an identity in the b_j, the coefficients of the various terms in the two members of the identity are equal.

If we expand the left member of (4), we find, as the coefficient of $b_{i_1} b_{i_2} \ldots . b_{i_r} b_0^{n-r}$, precisely $\Sigma x^{i_1}{}_1 x^{i_2}{}_2 \ldots . x^{i_r}{}_r$. In the right member, the coefficient of the same term is the product of K by a sum of determinant $D_1 + D_2 + \ldots . + D_1$ which involve the quantities a_j but which are independent of the b_j, that is,

$$\Sigma x^{i_1}{}_1 x^{i_2}{}_2 \ldots . x^{i_r}{}_r = K(D_1 + D_2 + \ldots . + D_1). \tag{5}$$

The determinants D_j may be found by expanding the Sylvester determinant successively by its first n rows and selecting the terms required. A shorter method will be suggested in the examples below.

To find the value of K, we note that this value is independent of the x_j. Hence, we may find K by putting $x_1 = x_2 = \ldots . = x_r = 1$, $x_{r+1} = x_{r+2} = \ldots . = x_n = 0$. The left member of (5) then reduces to $r!/\alpha!\beta! \ldots . \lambda!$, where α is the number of exponents i_1, i_2, etc. that are equal to i_1, β the number equal to $i_{\alpha+1}$, etc. In the right member, we have $a_1 = -r$, $a_2 = r(r-1)/2!$, $a_3 = -r(r-1)(r-2)/3!$, etc. If we make these substitutions in (5), the value of K is determined, and, if we now substitute this value of K into (5), we have the required expression for the given symmetric function in terms of the a_j, or, if one prefers, by substitution from (2), in terms of the elementary functions.

In a given problem, we take $m = i_1$, that is, equal to the order of the given symmetric function. We may also put equal to zero all the

b_j except $b_{i_1}, b_{i_2}, \ldots . b_{i_r}$, and b_0, which actually enter into the required term of the resultant. Furthermore, if we put

$$i_1+i_2+\ \ldots . +i_r=w$$

(so that w is the weight of the given function), it is obvious from homogeneity considerations that the right member of (5) does not contain any coefficient a_j such that $j>w$. Hence, if $n>w$, we may place $a_{w+1}=a_{w+2}=\ \ldots . =a_n=0$, and since $b_0\neq 0$, we may form the resultant, not of $f_1(x)$ and $f_2(x)$, but of $f'_1(x)=x^w+a_1x^{w-1}+\ \ldots +a_w$ and $f_2(x)$.

The above method of expressing a symmetric function in terms of the a_j is especially convenient for expressing the power sums Σx^t_1. As an example, let us express Σx^1_1 in terms of $a_1, a_2, \ldots , a_6$.

We first find the determinant of order twelve which defines the resultant of $f'_1(x)=x^6+a_1x^5+a_2x^4+\ \ldots . +a_6$ and $f_2(x)=b_6x^6+b_0$. To find the determinants D that occur in the coefficients of $b_6b^5_0$ in the expansion of this Sylvester's determinant, we first write the cofactor of the b_6 in each row. In each of these six cofactors, we put the remaining $b_6=0$, expand, successively, by its first five rows, and remove the factor b^5_0. The algebraic sum of the resulting determinants, multiplied by K, equals Σx^6_1, that is,

$$\Sigma x^6_1=K\left(\begin{vmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6\\ 1 & a_1 & a_2 & a_3 & a_4 & a_5\\ 0 & 1 & a_1 & a_2 & a_3 & a_4\\ 0 & 0 & 1 & a_1 & a_2 & a_3\\ 0 & 0 & 0 & 1 & a_1 & a_2\\ 0 & 0 & 0 & 0 & 1 & a_1\end{vmatrix} - \begin{vmatrix} 1 & a_2 & a_3 & a_4 & a_5 & 0\\ 0 & a_1 & a_2 & a_3 & a_4 & a_6\\ 0 & 1 & a_1 & a_2 & a_3 & a_5\\ 0 & 0 & 1 & a_1 & a_2 & a_4\\ 0 & 0 & 0 & 1 & a_1 & a_3\\ 0 & 0 & 0 & 0 & 1 & a_2\end{vmatrix}\right.$$

$$+\begin{vmatrix} 1 & a_1 & a_3 & a_4 & a_5 & 0\\ 0 & 1 & a_2 & a_3 & a_4 & 0\\ 0 & 0 & a_1 & a_2 & a_3 & a_6\\ 0 & 0 & 1 & a_1 & a_2 & a_5\\ 0 & 0 & 0 & 1 & a_1 & a_4\\ 0 & 0 & 0 & 0 & 1 & a_3\end{vmatrix} - \begin{vmatrix} 1 & a_1 & a_2 & a_4 & a_5 & 0\\ 0 & 1 & a_1 & a_3 & a_4 & 0\\ 0 & 0 & 1 & a_2 & a_3 & 0\\ 0 & 0 & 0 & a_1 & a_2 & a_6\\ 0 & 0 & 0 & 1 & a_1 & a_5\\ 0 & 0 & 0 & 0 & 1 & a_4\end{vmatrix}$$

$$\left.+\begin{vmatrix} 1 & a_1 & a_2 & a_3 & a_5 & 0\\ 0 & 1 & a_1 & a_2 & a_4 & 0\\ 0 & 0 & 1 & a_1 & a_3 & 0\\ 0 & 0 & 0 & 1 & a_2 & 0\\ 0 & 0 & 0 & 0 & a_1 & a_6\\ 0 & 0 & 0 & 0 & 1 & a_5\end{vmatrix} - \begin{vmatrix} 1 & a_1 & a_2 & a_3 & a_4 & 0\\ 0 & 1 & a_1 & a_2 & a_3 & 0\\ 0 & 0 & 1 & a_1 & a_2 & 0\\ 0 & 0 & 0 & 1 & a_1 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & a_6\end{vmatrix}\right)$$

If we put $x_1=1$, $x_2=x_3=\ldots\ldots=0$, we find that $K=1$. Hence, after further simplification, we have

$$\Sigma x^6_1=\begin{vmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 \\ 1 & a_1 & a_2 & a_3 & a_4 & a_5 \\ 0 & 1 & a_1 & a_2 & a_3 & a_4 \\ 0 & 0 & 1 & a_1 & a_2 & a_3 \\ 0 & 0 & 0 & 1 & a_1 & a_2 \\ 0 & 0 & 0 & 0 & 1 & a_1 \end{vmatrix} - \begin{vmatrix} a_1 & a_2 & a_3 & a_4 & a_6 \\ 1 & a_1 & a_2 & a_3 & a_5 \\ 0 & 1 & a_1 & a_2 & a_4 \\ 0 & 0 & 1 & a_1 & a_3 \\ 0 & 0 & 0 & 1 & a_2 \end{vmatrix}$$

$$+\begin{vmatrix} a_1 & a_2 & a_3 & a_6 \\ 1 & a_1 & a_2 & a_5 \\ 0 & 1 & a_1 & a_4 \\ 0 & 0 & 1 & a_3 \end{vmatrix} - \begin{vmatrix} a_1 & a_2 & a_6 \\ 1 & a_1 & a_5 \\ 0 & 1 & a_4 \end{vmatrix} + \begin{vmatrix} a_1 & a_6 \\ 1 & a_5 \end{vmatrix} - a_6$$

In similar way, it is seen that Σx^t_1 may be expressed as a sum of t determinants of maximum order t.

As a second example, let us find the expression for $\Sigma x^4_1x^3_2$. We write the determinant, of order eleven, which defines the resultant of $x^7+a_1x^6+\ldots\ldots+a_7$ and $b_4x^4+b_3x^3+b_0$. In it, we form the cofactor of each b_4 and put $b_4=0$ in each result. In each of the resulting determinants, we form the cofactor (if it is not zero) of each b_3 and put $b_3=0$ in each result. We then form the successive cofactors of the b_0 in the resulting determinants and arrive, in this way, at the determinants D. We have

$$\Sigma x^4_1x^3_2=-K\left(\begin{vmatrix} a_1 & a_3 & a_4 & a_5 \\ 1 & a_2 & a_3 & a_4 \\ 0 & a_1 & a_2 & a_3 \\ 0 & 1 & a_1 & a_2 \end{vmatrix} - \begin{vmatrix} a_1 & a_2 & a_4 & a_6 \\ 1 & a_1 & a_3 & a_5 \\ 0 & 1 & a_2 & a_4 \\ 0 & 0 & a_1 & a_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_3 & a_7 \\ 1 & a_1 & a_2 & a_6 \\ 0 & 1 & a_1 & a_5 \\ 0 & 0 & 1 & a_4 \end{vmatrix}\right.$$

$$+\begin{vmatrix} 1 & a_2 & a_5 & a_6 \\ 0 & a_1 & a_4 & a_5 \\ 0 & 1 & a_3 & a_4 \\ 0 & 0 & a_2 & a_3 \end{vmatrix} - \begin{vmatrix} 1 & a_2 & a_3 & 0 \\ 0 & a_1 & a_2 & a_7 \\ 0 & 1 & a_1 & a_6 \\ 0 & 0 & 1 & a_5 \end{vmatrix} - \begin{vmatrix} 1 & a_3 & a_4 & a_6 \\ 0 & a_2 & a_3 & a_5 \\ 0 & a_1 & a_2 & a_4 \\ 0 & 1 & a_1 & a_3 \end{vmatrix}$$

$$+\begin{vmatrix} 1 & a_1 & a_3 & 0 \\ 0 & 1 & a_2 & 0 \\ 0 & 0 & a_1 & a_7 \\ 0 & 0 & 1 & a_6 \end{vmatrix} + \begin{vmatrix} 1 & a_2 & a_4 & a_7 \\ 0 & a_1 & a_3 & a_6 \\ 0 & 1 & a_2 & a_5 \\ 0 & 0 & a_1 & a_4 \end{vmatrix} - \begin{vmatrix} 1 & a_1 & a_5 & a_7 \\ 0 & 1 & a_4 & a_6 \\ 0 & 0 & a_3 & a_5 \\ 0 & 0 & a_2 & a_4 \end{vmatrix}$$

$$-\begin{vmatrix} 1 & a_1 & a_2 & 0 \\ 0 & 1 & a_1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & a_7 \end{vmatrix} - \begin{vmatrix} 1 & a_2 & a_3 & 0 \\ 0 & a_1 & a_2 & a_7 \\ 0 & 1 & a_1 & a_6 \\ 0 & 0 & 1 & a_5 \end{vmatrix} + \begin{vmatrix} 1 & a_1 & a_3 & 0 \\ 0 & 1 & a_2 & 0 \\ 0 & 0 & a_1 & a_7 \\ 0 & 0 & 1 & a_6 \end{vmatrix}$$

$$\left. - \begin{vmatrix} 1 & a_1 & a_2 & 0 \\ 0 & 1 & a_1 & a \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & a_7 \end{vmatrix} \right)$$

If we put $x_1 = x_2 = 1$, $x_3 = x_4 = \ldots = 0$, we find that $K = 1$. Hence

$$\Sigma x^4{}_1 x^3{}_2 = - \begin{vmatrix} a_1 & a_3 & a_4 & a_5 \\ 1 & a_2 & a_3 & a_4 \\ 0 & a_1 & a_2 & a_3 \\ 0 & 1 & a_1 & a_2 \end{vmatrix} + \begin{vmatrix} a_1 & a_2 & a_4 & a_6 \\ 1 & a_1 & a_3 & a_5 \\ 0 & 1 & a_2 & a_4 \\ 0 & 0 & a_1 & a_3 \end{vmatrix} - \begin{vmatrix} a_1 & a_2 & a_3 & a_7 \\ 1 & a_1 & a_2 & a_6 \\ 0 & 1 & a_1 & a_5 \\ 0 & 0 & 1 & a_4 \end{vmatrix}$$

$$- \begin{vmatrix} a_1 & a_4 & a_5 \\ 1 & a_3 & a_4 \\ 0 & a_2 & a_3 \end{vmatrix} + 2 \begin{vmatrix} a_1 & a_2 & a_7 \\ 1 & a_1 & a_6 \\ 0 & 1 & a_5 \end{vmatrix} + \begin{vmatrix} a_2 & a_3 & a_5 \\ a_1 & a_2 & a_4 \\ 1 & a_1 & a_3 \end{vmatrix} - \begin{vmatrix} a_1 & a_3 & a_6 \\ 1 & a_2 & a_5 \\ 0 & a_1 & a_4 \end{vmatrix}$$

$$-2 \begin{vmatrix} a_1 & a_7 \\ 1 & a_5 \end{vmatrix} + \begin{vmatrix} a_3 & a_5 \\ a_2 & a_4 \end{vmatrix} + 2a_7.$$

As a final example, let us find the expression for $\Sigma x^3{}_1 x^3{}_2 x_3 x_4$. We find the determinant, of order eleven, which defines the resultant of $x^8 + a_1 x^7 + \ldots + a_8$ and $b_3 x^3 + b_1 x + b_0$. We form, in all possible ways, the successive cofactors of two b_3 and put $b_3 = 0$ in the resulting determinants; in each of these, the successive non-zero cofactors of two b_1 and put the remaining $b_1 = 0$; and, finally, the successive cofactors of the remaining four b_0. By putting $x_1 = x_2 = x_3 = x_4 = 1$, $x_5 = x_6 = \ldots = 0$ we find that $K = 1$. The result is

$$\Sigma x^3{}_1 x^3{}_2 x_3 x_4 = \begin{vmatrix} a_2 & a_3 & a_6 \\ a_1 & a_2 & a_5 \\ 1 & a_1 & a_4 \end{vmatrix} - \begin{vmatrix} a_1 & a_4 & a_6 \\ 1 & a_3 & a_5 \\ 0 & a_2 & a_4 \end{vmatrix} - \begin{vmatrix} a_1 & a_3 & a_7 \\ 1 & a_2 & a_6 \\ 0 & a_1 & a_5 \end{vmatrix} + 3 \begin{vmatrix} a_1 & a_2 & a_8 \\ 1 & a_1 & a_7 \\ 0 & 1 & a_6 \end{vmatrix}$$

$$+ \begin{vmatrix} a_4 & a_5 \\ a_3 & a_4 \end{vmatrix} + 2 \begin{vmatrix} a_3 & a_6 \\ a_2 & a_5 \end{vmatrix} - 2 \begin{vmatrix} a_2 & a_7 \\ a_1 & a_6 \end{vmatrix} - 3 \begin{vmatrix} a_1 & a_8 \\ 1 & a_7 \end{vmatrix} + 6a_5. \qquad (6)$$

It may, perhaps, be noted further that, when the expression for a given symmetric function has been found by the above method, the

determinant form in which the result appears is a convenient one from which to determine also the expression for any symmetric function defined by omitting one or more letters from the type term of the given function.

In the last illustration given above, for example, the coefficient of x^3_n is $\Sigma^1 x^3_1 x_2 x_3$, wherein Σ^1 extends over $x_1, x_2, \ldots . x_{n-1}$. Moreover

$$a_j = a'_j - a'_{j-1} x_n$$

wherein a'_j and a'_{j-1+n} are coefficients in the equation whose roots are $x_1, x_2, \ldots . x_{n-1}$.

If we substitute the values of the a_j from (7) into (6) and equate the coefficients of x^3_n on the two sides of the resulting equation, we have, after simplifying and dropping the primes,

$$\Sigma x^3_1 x_2 x_3 = -\begin{vmatrix} a_1 & a_2 & a_5 \\ 1 & a_1 & a_4 \\ 0 & 1 & a_3 \end{vmatrix} + \begin{vmatrix} a_2 & a_4 \\ a_1 & a_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_5 \\ 1 & a_4 \end{vmatrix} - 3a_5.$$

Since n may be as large as we please, this is the general expression for this function.

Similarly, if we equate the coefficients of the first power of x_n on the two sides of equation (6), we obtain

$$\Sigma x^3_1 x^3_2 x_3 = -\begin{vmatrix} a_2 & a_3 & a_6 \\ a_1 & a_2 & a_5 \\ 0 & 1 & a_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_4 & a_6 \\ 1 & a_3 & a_5 \\ 0 & a_1 & a_3 \end{vmatrix} + \begin{vmatrix} a_1 & a_3 & a_7 \\ 1 & a_2 & a_6 \\ 0 & 1 & a_4 \end{vmatrix} - 3a_5 \begin{vmatrix} a_1 & a_2 \\ 1 & a_1 \end{vmatrix}$$

$$-\begin{vmatrix} a_4 & a_5 \\ a_2 & a_3 \end{vmatrix} - 2\begin{vmatrix} a_3 & a_6 \\ a_1 & a_4 \end{vmatrix} + 2\begin{vmatrix} a_2 & a_7 \\ 1 & a_5 \end{vmatrix} + 3a_1 a_6 - 6a_7.$$

THE POET AND THE MATHEMATICIAN

I never yet encountered the mere mathematician who could be trusted out of equal roots, or one who did not clandestinely hold it as a point of his faith that x^2+px was absolutely and unconditionally equal to q. Say to one of these gentlemen that you believe that occasions may occur when x^2+px is not altogether equal to q and get out of his reach as speedily as convenient for beyond doubt he will endeavor to knock you down—From Edgar Allen Poe in "Purloined Letter."

A Special Case of a Transformation Due to Gurney

By L. J. Adams
Santa Monica Junior College, California

Dr. L. E. Gurney has applied a simple transformation to a polynomial of the type $a_0x^n+a_1x^{n-1}+a_2x^{n-2}+\ldots+a_{n-1}x$ with the result that a new polynomial of degree $n-1$ is obtained.

The following is a special case of this transformation:

Consider the relation

$$y=a\,x^n$$

Apply the transformation:

$$y=Y\,(X-1)^{n-1}$$
$$x=X-1$$

The equation becomes

$$Y=a\,(X-1)$$

which is a straight line.

This fact can sometimes be used advantageously to test a set of data. The work may be displayed as follows:

Given data

x	0	1	2	3	4	4	5
y	0	1	8	27	27	64	125

In this case the original data obey the law $y=a\,x^3$

Apply the transformation:

$$y=Y\,(X-1)^2$$
$$x=X-1$$

The transformed data are:

X	1	2	3	4	5	6
Y	0	1	2	3	4	5

Note that the X's are each one integer larger than the corresponding x's. The Y's are computed by means of the relation $Y=\frac{y}{x^2}$, which follows directly from the equations of the transformation.

If the transformed points comprise a straight line when plotted, the original data represented a cubic of the form $y=a\,x^3$. The slope of the straight line is equal to the value of "a" in the equation of the original cubic, and therefore can be employed to determine this coefficient.

The Teachers Department

Edited by

JOSEPH SEIDLIN AND W. PAUL WEBBER

THE PRACTICAL MATHEMATICS BOOK

While rearranging some books in my library I came upon a little volume published several years ago which treats what may be called practical mathematics. In particular it treats what is called shop mathematics. In glancing through it I noticed some of its methods and definitions. Naturally there is a distinct effort at informality of statement of definitions and principles. In itself this is no pedagogical sin but rather to be commended in a book of its kind. However, quite a number of the definitions are, by implication, either somewhat misleading, or else they are of so restricted a character as to be of little use beyond the book, and therefore not well adapted as a basis for further study in mathematics. If this were necessary in carrying out the purpose of the author, it could be justified. It is here that I take issue with his method. It seems better to give the correct and accepted definitions of the science when this can be just as easily done.

For example, a parallelopiped is defined as a six-sided solid all of whose faces are parallelograms. Here is a confusion of the terms side and face. This will be a source of misunderstanding if the pupil is ever in later life to converse with mathematicians or scientists on the topics concerned. The same may be said if he is ever to make public statements either verbal or written on these topics. Of course the mechanically inclined will get the author's meaning in so far as the terms are used in this book. But is that a sufficient objective? Again, a negative number is defined as one to be subtracted. Then is 3 a negative number in $7-3=4$? The same definition goes on further and says a negative number lies to the left of zero on the number scale. It is true that the author has gone to some length in the context to develop the idea of the number scale. Further the term index is defined separately in two different senses. One says an index is a number to the left and above a radical. This is true enough for the instance at hand. But would it not be better to define index of a radical and not leave the impression that index is so restricted in its meaning. The other definition refers to a mechanical idea and is subject to the same objection. One could go on indefinitely pointing out such "loose" statements in what is otherwise apparently a very good and useful little book. The case is similar to

that of the old definition of limit as found in the high school texts of a generation ago. In that case it was the "exception clause" of the definition that caused the difficulty. Again in this classic case the correct statement is just as easy as the faulty or restricted one. But there is a vast difference in the two when it comes to the study of mathematics a little beyond the high school.

There are small manuals of mathematical matter allied to other fields such as home economics, electrical engineering, elementary accounting, etc., treated from the practical and intuitive standpoint. These all have a legitimate place in education. They would fit in at the end of the grade school or at the beginning of high school and would form an excellent background as a requirement for the formal courses in mathematics. The choice of course would not be of great importance, if the pupils are interested for the time being. Much material is common to all such courses. This background would be an excellent basis for the generalizations and idealizations required in the formal courses. Those not going on to the formal courses will have a useful modicum of mathematics for every day life. That such subject matter should be given now in school whereas formerly much of it was picked up by the pupil in his home life, is merely an incident to the changed environment of average pupils now as compared to a generation ago. Many schools practice such a regime as suggested above and we can only commend them and hope that some such plan will soon be universal.

W. PAUL WEBBER

DE MORGAN ON GEOMETRY STUDY

But, although there is no study which presents so simple a beginning as that of geometry, there is none in which difficulties grow more rapidly as we proceed, and what may appear at first rather paradoxical, the more acute the student the more serious will the impediments in the way of his progress appear. This necessarily follows in a science which consists of reasoning from the very commencement, for it is evident that every student will feel a claim to have his objections answered, not by authority but by argument, and that the intelligent student will perceive more readily than another the force of an objection and the obscurity arising from an unexplained difficulty.—From "On the Study of Mathematics."

Notes and News Department

Edited by
I. MAIZLISH

The Editor of this Department desires to again appeal to each and every reader of the *National Mathematics Magazine*, as well as to the Heads of the Mathematics Departments of the various institutions reached by the Magazine, to send him all items of interest. Please understand that this is the ONLY means available to the Editor for securing material for this Department. He cannot create news items for this Department. Will you not cooperate with him in making the *National Mathematics Magazine* just a bit more interesting?

The Editor takes this means of expressing his appreciation of the cooperation shown by some members of the editorial staff in sending in news items.

Mr. J. K. Upchurch has been appointed Instructor in mathematics at Mississippi State College.

Mr. S. B. Murray has returned to the mathematics department of Mississippi State College after completing requirements for the Master's Degree at the University of Chicago.

The Kappa Mu Epsilon, Mathematical Fraternity, is sponsoring a mathematical exhibit at Mississippi State College under the supervision of the Mathematics Department. Problems of historic interest and problems of practical importance will be drawn on large placards suitable for exhibit and for use in classes. Members of the local Chapter have suggested a variety of problems.

Professor A. E. Whitford, who was co-head of the Department of Mathematics at Alfred University, has been promoted to the deanship of the college of Liberal Arts; Professor Joseph Seidlin, who was also co-head of the department, is now the departmental executive.

The second annual meeting of the Wisconsin Section of the Mathematical Association of America was held at the Oshkosh State Teachers College on Saturday, May 5th, 1934. The chief address of the session was on "Modern Trends and Perils in Mathematics" by Professor E. B. Skinner of the University of Wisconsin. The following officers were elected for the year 1934-35: Chairman, Professor H. P.

Pettit, Marquette University; Secretary, Professor May M. Beenken, Oshkosh State Teachers College; Program Committee, Professor E. B. Skinner, University of Wisconsin and Professor Irene Price, Oshkosh State Teachers College. It was suggested that at the next annual meeting, to be held at Marquette University, the central theme for the session should be the building up of a course in the teaching of mathematics which would be valuable as a course in mathematics as well as a course in pedagogy.

The mathematics club of the Oshkosh State Teachers College is planning to further interest in secondary mathematics by presenting programs before the local high school mathematics club.

Dr. W. V. Parker, formerly Professor and Head of the Department of mathematics at the Mississippi State Teachers College, has recently been appointed to an Assistant Professorship at the Georgia School of Technology. Dr. Parker was a member of the Executive Committee of the Southern Intercollegiate Mathematics Association.

Dr. B. E. Mitchell of Millsaps College is Chairman of a Committee appointed by the mathematics section of the Mississippi Education Association. This committee selected a standard test of mathematical training to be given to college freshmen throughout the State. Many colleges are cooperating.

The honorary degree of doctor of science was conferred on Founder's Day by Lehigh University on Dr. Irving Langmuir, associate director of the Research Laboratory of the General Electric Company.

The degree of LL.D. will be conferred on General J. C. Smuts on the occasion of his installation as rector of the University of St. Andrews on October 17. The degree will also be conferred on Sir Thomas Holland, principal and vice-chancellor of the University of Edinburgh, president of the British Association for the advancement of Science in 1929, and on John Hutchinson, of the Kew Herbarium.

Professor Charles A. Corcoran has been made head of the department of physics of the College of the City of New York, succeeding Professor William Fox, who retired with the title of professor emeritus.

Drs. Harlow Shapley, Donald H. Menzel, and Loring B. Andrews, of the staff of the Harvard Observatory, were recently appointed members of the board of regents of "The Telescope," a new popular

illustrated magazine of astronomy, published by the Bond Astronomical Club with the cooperation of the staff of the observatory.

Norman Taylor, editor for botany of the new Webster's Dictionary, has become editorial and promotion adviser in the field of natural science and outdoor life for the Houghton-Mifflin Company. He will carry on his work at the New York office of the publishers.

Dr. Wolfgang Kohler, professor of philosophy and director of the Psychological Institute at the University of Berlin, is the first psychologist to be appointed William James lecturer in philosophy and psychology at Harvard University. His predecessors are Professors John Dewey and Arthur O. Lovejoy. There will be ten lectures, at five o'clock, on Tuesday afternoons, from October 9 to December 11. Professor Kohler has chosen for the title of his lectures: "Beyond Psychology: Psychology and the Study of Nature."

According to an Associated Press dispatch work has started on the Medical center at Shanghai. The center will include a medical college and a hospital. The site, consisting of twenty-one acres in the French concession, was purchased nearly ten years ago by the Rockefeller Foundation for $440,000.00. Its value is said to be now a million and a half dollars.

The Southern Intercollegiate Mathematics Association, which was organized at Centenary College in October, 1933, will hold its second annual meeting in May, 1935, at Centenary College, Shreveport, La. It is hoped that additional institutions will participate in this year's contests. Any institution in the States of Louisiana, Arkansas, Texas, Mississippi, and Oklahoma may join the S. I. M. A. by writing to the secretary, Miss Frances White, Louisiana Polytechnic Institute, Ruston, Louisiana.

GAUSS ON ARITHMETIC

"The most beautiful theorems of higher mathematics have this peculiarity, that they are easily discovered by induction, while on the other hand their demonstrations lie in exceeding obscurity and can be ferreted out only by very searching investigations. It is precisely this which gives to higher arithmetic that magic charm which has made it the favorite science of leading mathematicians."

Book Review Department

Edited by
P. K. Smith

A History of Mathematics in America Before 1900. By David Eugene Smith and Jekuthiel Ginsburg. Chicago, The Open Court Publishing Company, 1934. 209 pages.

This little volume is No. 5 of The Carus Monograph Series, in which it appears as "the first one of a purely historical character".

The subject is a difficult one to treat satisfactorily and the Carus Monograph Committee are certainly to be congratulated upon their selection of persons to do the work. All American lovers of mathematics will be pleased with the results.

The word *"America"* as used in the title of the book includes "territory north of the Rio Grande River and the Caribbean Sea". The time covered is treated in four chapters: "The Sixteenth and Seventeenth Century", "The Eighteenth Century", The Nineteenth Century," and "The Period 1875-1900". In treating the first period, it is revealed that, from 1500 to 1600, the level of mathematics was below that of the elementary work of a grammar school and that, from 1600 to 1700, it was not even equal to that of a low grade high school. Before 1700, America produced no scientists of note—it was all *new* and had very little need of science except a calendar, a little astronomy for navigation, and a little arithmetic for trading and surveying. Training for the clergy predominated. However, it saw the founding of Harvard (1636) and William and Mary (1693). Also, in 1660, the Virginia Assembly enacted legislation looking toward the establishment of a college and free school.

Only fourteen pages are given to this period.

The second chapter reveals that by the time of the Revolution the Colonies had ten colleges, and that the general nature of the work in America was found in the two great universities of England but that it lacked quality. In science, the tendency was still toward applications to astronomy.

The chapter begins with a general survey of the period. Then follows a list of the colleges of the time (page 18), and a discussion of their work. A list of nine theses from those of Yale for the year 1718 is given to show the nature of the subjects studied at that time. Notice is taken of "private instruction", "equipment for study", "text books" and "societies and periodicals". Special notice is given John Winthrop

(1714-1779), David Rittenhouse (1732-1796), Benjamin Franklin (1706-1790), and Thomas Jefferson (1743-1826).

It is very interesting to know the part played by so many of those who surveyed or helped to survey the vast wilderness then peculiar to America. Many of these men later became governors, judges, generals, and recognized scientists.

A good summary of the period closes this chapter of fifty pages.

Pages 65-101 covers the third period, 1800-1875, years which "gave evidence of an awakening".

Again the authors give a general survey of the period. Under the heading "The Colleges and Universities", these institutions are defined as they then existed. At the beginning of the 19th century, Harvard had no entrance requirements; "not until 1837 was arithmetic dropped from the freshman course"; "the curriculum was rigidly fixed". As samples, the curricula in mathematics as taught at Dartmouth and Yale in 1825 and 1850 respectively are given (page 71). In mathematics, astronomy was still the principal study. At Harvard, from 1782 to 1836, 68 per cent of the mathematics theses for the last two years of the college course were on astronomy.

Libraries, European influence, scientific societies and periodicals are each briefly treated. Editors of present-day periodicals will find encouragement from reading pages 83-91. Under the heading "Prominent Names", 1800-1875, are found Robert Adrian (b. Ireland, 1775). Nathaniel Bowditch (b. Salem, Mass., 1773), William B. Rogers (b. Philadelphia, 1804), Theodore Strong (b. Hadley, Mass., 1790), Charles Gill (b. England, 1805), Alexander Dallas Bache (b. Philadelphia, 1806), and Ferdinand Rudolph Hassler (b. Switzerland, 1770).

The last period, 1875-1900, because of its greater importance is given the greatest amount of space—the last half of the whole volume. In this period, mathematics became a 'subject within itself rather than a minor for physics and astronomy'. It was the period of research, very revolutionary in its nature. Three great forces contributed powerfully to this reform: (1) The vision of men like Gilman, president of John Hopkins 1875-1901, and Eliot, president of Harvard 1869-1909, (2) The American Mathematical Society, and (3) European Influence. The full significance of each of these three influences can be appreciated only after reading the details of their history.

Other sub-chapters of Chapter IV are "Periodicals", "Prominent Names" and "Special Interests", "American Dissertations", "General Trend 1875-1900", "Trend of Important Branches", and "Retrospect". The reader will be amazed with the sharp contrast of the almost exclu-

sive interest in astronomy of the earlier periods and the active interest in each of the twelve special subjects presented by the authors: (1) Algebra, (2) Function Theory, (3) Quantics, (4) Transformations, (5) Calculus, (6) Differential Equations, (7) Theory of Numbers, (8) Theory of Groups, (9) Determinants, (10) Quaternions and Vector Analysis, (11) Probability and Methods of Approximation, and (12) Geometry.

Notice is also made of the interest shown during this period by foreign contributors and foreign journals.

The volume is made more attractive by the inclusion of photographs of a number of the more prominent mathematicians of the periods treated. John Winthrop (1714-1779) is the first in the list; E. H. Moore (1862-1932) is also in the list. A splendid eight-page index is added.

In closing, the authors very fittingly raise the question as to the future: In the fourth period mathematics was "pure" mathematics, except for such subjects as Quaternions and Differential Equations, but earlier there was a strong tendency toward applications. Which tendency is better for mathematics, or for the natural sciences? Has the tendency again turned toward applications?

In conclusion, the present reviewer wishes to express the genuine inspiration he has derived from reviewing this little volume; he is certain that its influence will fully justify the wisdom of the committee who conceived the piece of work and the authors who have so well executed it.

IRBY C. NICHOLS

INFINITY

Nor can the existence of the infinite be established mathematically because infinity, the inexhaustibility of the counting process, is a mathematical assumption, *the basic assumption of arithmetic*, on which all mathematics rests. Is it then a supernatural truth, one of those few gifts which the Creator bestowed upon man when he cast him into the universe, naked and ignorant, but free to shift for himself? Or has the concept of infinity grown upon man, grown out, indeed, out of his futile attempts to reach the last number.—From Tobias Dantzic in "Number the Language of Science."

Problem Department

Edited by

T. A. Bickerstaff

This department aims to provide problems of varying degrees of difficulty which will interest anyone who is engaged in the study of mathematics.

All readers, whether subscribers or not, are invited to propose problems and to solve problems here proposed.

Problems and solutions will be credited to their authors.

While it is our aim to publish problems of most interest to the readers, it is believed that regular text-book problems are, as a rule, less interesting than others. Therefore, other problems will be given preference when the space for problems is limited.

Send all communications about problems to T. A. Bickerstaff, University, Mississippi.

SOLUTIONS

No. 61. Proposed by A. F. Moursund, University of Oregon, Eugene, Oregon.

Show that

$$\sum_{i=2}^{P+1} \frac{(-1)^i(P+1)!}{i(i-2)!\,(P+1-i)!} = 1.$$

Solved by G. W. Petrie, Carnegie Institute of Technology.

$$\sum_{i=2}^{P+1} \frac{(-1)^i(P+1)!}{i\,(i-2)!(P+1-i)!} =$$

$$\sum_{i=0}^{P+1} (-1)^i(i-1)\binom{P+1}{i} - \sum_{i=0}^{1} (-1)^i(i-1)\binom{P+1}{i} =$$

$$\sum_{i=0}^{P+1} (-1)^i i\binom{P+1}{i} - \sum_{i=0}^{P+1} (-1)^i\binom{P+1}{i} - (-1)+0 =$$

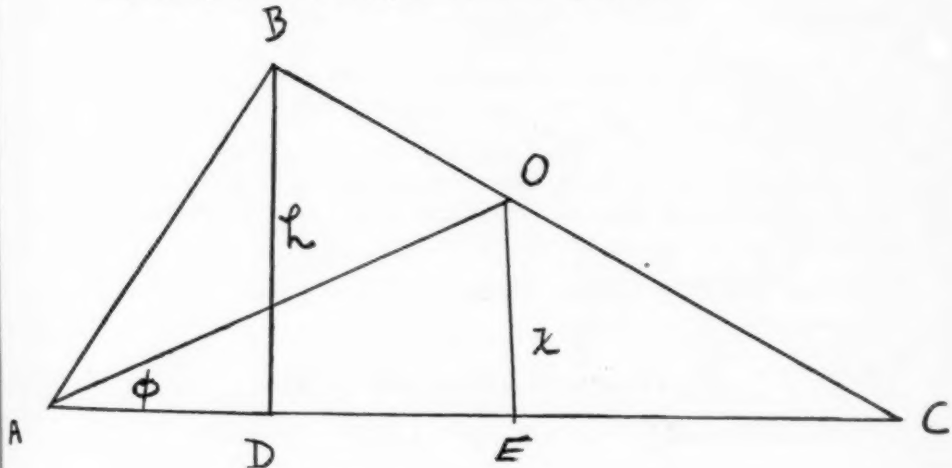
$$\sum_{i=0}^{P+1} (-1)^i(P+1)\binom{P-1}{i} - (1-1)^{P+1}+1 =$$

$$(P+1)(1-1)^P - 0 + 1 = 1$$

No. 64. Proposed by C. L. Wilson, Prairie View State Normal and Industrial College, Prairie View, Texas.

Given an oblique triangle ABC, with sides opposite, abc, b is the base, h is the altitude, and A (equal to Θ) is a base angle. From A, a line is drawn cutting side a at O, forming a triangle AOC, whose base is b and whose altitude is x. If h/x equals r, find an expression for angle AOC in terms of h, x, Θ, r, and b.

Solved by W. B. Clarke, and T. A. Bickerstaff.



$$DC/EC = h/x = r$$

$$EC = DC/r$$

$$AE = b - EC$$

$$= b\gamma - DC/r$$

$$\Phi = \arctan x/AE$$

$$= \arctan xr/br - DC$$

$$= \arctan h/br - DC$$

Now $\quad DC = O\ b - AD$

$$= b - h \cot \Theta$$

Therefore, $\quad \Phi = \arctan h/b(r-1) + h \cot \Theta$

No. 65. Proposed by W. B. Clarke.

Considering only triangles whose sides are consecutive integers, and whose area is an integer, find the area of the triangle which is next larger than one whose area is 16296.

Solution by E. P. Starke, Rutgers University, New Brunswick, N. J.

Taking for sides the integers $a-1$, a and $a+1$, the area is given by $S = a\sqrt{3(a^2-4)}/4$. For S to be also an integer, we must have a an even integer—say $a = 2y$. Then also $3(y^2-1)$ must be a perfect square—say $3(y^2-1) = 9x^2$. We have now $y^2-1 = 3x^2$; $S = 3xy$; $2y-1$, $2y$, $2y+1$ are the sides.

All solutions of $y^2-1 = 3x^2$ are given by the following reduction formulae:

$X_{n+1} = 2x_n + y_n$, $y_{n+1} = 3x_n + 2y_n$, where $x_0 = 0$, $y_0 = 1$. Hence we compute: $x_1 = 1$, $y_1 = 2$; $x_2 = 4$, $y_2 = 7$; $x_3 = 15$, $y_3 = 26$; $x_4 = 56$, $y_4 = 97$; $x_5 = 209$, $y_5 = 362$; etc.

Evidently 4_4 and y_4 produce the triangle cited in the statement of the problem; for the next larger triangle, x_5 and y_5 give an area of 226,974.

Also solved by Hobson M. Zerbe, Wilkes-Barre, Penn., and the proposer.

PROBLEMS FOR SOLUTION

No. 66. Proposed by H. T. R. Aude, Colgate University.

An ellipse with a major axis $2a$ and a minor axis $2b$ is circumscribed by a parabolic arc and a chord of the parabola, perpendicular to its axis. If the axis of the parabola contains the major axis of the ellipse, show that the minimum area of the parabolic segment is $9ab/2$.

No. 67. Proposed by Walter B. Clarke, San Jose, Cal.

Let A equal area of a triangle

A_0 equal area of its orthic triangle

P equal product of the sides of the given triangle

P_0 equal product of the sides of its orthic triangle

To show that $A_0 \cdot P$ equals $2A \cdot P_0$.

No. 68. Proposed by Walter B. Clarke, San Jose, Cal.

To prove that the product of the alternate (non-adjacent) segments of the sides of a triangle as cut by altitudes equals the product of the sides of its orthic triangle.

[1]The complete solution in integers of the Diophantine equation $y^2 = (a^2 \pm 1)x^2 \pm 1$ is given as problem 3677 in the American Mathematical Monthly. This will appear in print in due time.